Effect of anisotropic permeability on the dynamic response of cartilage under nanoindentation

P. Vena¹ and D. Gastaldi²

¹ Department of Chemistry, Materials and Chemical Engineering, Politecnico di Milano, Italy
email: pasquale.vena@polimi.it; dario.gastaldi@polimi.it

Abstract— Dynamic nanoindentation is an experimental technique suitable to characterize articular cartilage on its superficial layer and to determine elastic properties as well as hydraulic tissue permeability. Anisotropic mechanical properties of the tissue, owed to its micro-scale architecture of the collagen fibril network, makes the mechanical characterization a challenging task. Models for the dynamic nanoindentation experiment provide useful insight allowing for more refined characterization technique. In this paper, dynamic nanoindentation of cartilage is investigated with particular reference to the tissue anisotropic mechanical and physical properties. Anisotropic permeability is introduced in a coupled fluid-mechanics modeling in the frequency domain. The focus of this work is on the effect of anisotropy in tissue permeability on the frequency response of the cartilage. The final aim is to define specific experimental measures able to identify hydraulic permeability properties of the tissue. To this aim, the peculiar effect of geometric features (indentor radius) is investigated.

Keywords—cartilage nanoindentation, frequency response, anisotropic permeability.

I. INTRODUCTION

The analysis of articular cartilage properties is a branch of great interest in both clinical and biomaterials study; in fact, its peculiar structure strongly inhomogeneous, anisotropic, non-linear, with time and depth-dependent features [1] gives its fundamental role in the correct functionality of joints, in terms of load transmission and lubrication support. This tissue can be seen as a porous solid matrix, constituted by chondrocytes, collagen fibers and proteoglycans, saturated with synovial liquid; therefore, mechanisms that rule its mechanical behaviour are viscoelasticity, due to the solid phase, and poroelasticity, due to the fluid flow through the pores. In particular, the superficial layer has a very important role; acting as a protective barrier, its damage involves decreasing in stiffness, with resulting problems in load transmission, and increasing in permeability, altering its physiological internal pressurization [2]. Therefore, the study of properties like stiffness and permeability is crucial for establishing a correlation between their variations and pathological condition of the overall tissue. For the simulation of cartilage actual load we use dynamic mechanical analysis, where the load is imposed cyclically [3]. Through indentation we can get the contribution of single constituents at nanometric level. Therefore, in this work, we simulate and analyse, exploiting a combination of the finite element framework Abaqus (Simulia, Providence, RI, USA) and the data processing software Matlab (MathWorks, Natick, MA, USA), the frequency response of materials tested in indentation, evaluating how anisotropic parameters as stiffness and permeability affect superficial layer mechanical behaviour.

II. METHODS

Dynamic nanoindentation tests are here simulated. In this experiment, a spherical tip is put in contact with the sample and a preload is applied. A harmonic oscillation at a given frequency is applied at the tip and the force is monitored throughout the cycles. When stationarity oscillation is achieved for a given frequency, force oscillation amplitude and time shifts are recorded. The measurement is performed for a wide range of frequency so to have a spectrum of the frequency response of the tissue [4]. The dynamic nanoindentation experiments have been simulated in the frequency domain by simulating the coupled mechanical response of a fully saturated deformable anisotropic material [5]. Given the small harmonic oscillations of the indenter tip, a linearized anisotropic stress-strain relationship has been used:

\[
\Delta \varepsilon = \begin{bmatrix} \lambda & -\nu & -\lambda\eta & 0 \\ -\nu & 1 & -\nu & 0 \\ -\lambda\eta & -\nu & \lambda & 0 \\ 0 & 0 & 0 & \frac{ka}{\alpha} \end{bmatrix} \Delta \sigma \quad (1)
\]

in which \(\lambda\) is the elastic anisotropy ratio \(E_a/E_r\) (axial versus radial elastic modulus), \(\eta\) is the out-of-isotropy plane Poisson ratio and \(\nu\) is the in-plane Poisson ratio.

The fluid flow through the porous microstructure has been modeled through the Darcy type relationship:

\[
q = K \nabla p \quad (2)
\]

in which \(K\) is the permeability tensor that for anisotropic tissue is here assumed as:

\[
K = \begin{bmatrix} k_a & 0 \\ 0 & k_r \end{bmatrix} \quad (3)
\]

and \(\nabla p\) is the fluid pressure gradient.

In order to perform a parametric study, anisotropy ratio for the permeability \(A_k = k_a/k_r\) and an average permeability \(K = \frac{k_a + 2k_r}{3}\) have been introduced.

The linearized problem in the frequency domain is the solved through the linear finite element system of equations:

\[
\begin{bmatrix} K_{uu} & -G_{up} \\ -fG_{pa} & -K_{pp} \end{bmatrix} \begin{bmatrix} U \\ P \end{bmatrix} = \begin{bmatrix} F_u \\ F_p \end{bmatrix} \quad (4)
\]
in which \( U \) and \( P \) are the nodal variables of displacement and fluid pressure, respectively, \( f \) is the frequency of excitation of the tip and \( i \) is the imaginary unit.

In this study four different tip radii have been simulated: 400 \( \mu \)m, 125 mm, 25 \( \mu \)m and 7.5 \( \mu \)m. The initial penetration depth (pre-load) was such that \( h_0=R/10 \) in all cases so to have the same strain level in the tissue for all the studied cases. The finite element model in the frequency domain provided as results the force amplitude and the time shift for all frequency studied.

**III. RESULTS AND DISCUSSIONS**

Fig. 1 shows the typical spectrum for the time shift in a dynamic nanoindentation simulations with a 7.5 \( \mu \)m tip radius and different permeability anisotropy ratios (average permeability is kept constant). It is worth noting that permeability anisotropy has an appreciable effect on two main experimental factors: i) the maximum time shift (peak of the curves) and ii) the frequency at maximum time shift.

Fig. 2 shows the value of the maximum time shift vs permeability anisotropy ratio \( (A_k) \) for different tip radii. Time shift increases with \( A_k \) (i.e. with increasing axial permeability with respect to radial permeability). Fig. 3 shows the frequency at peak vs \( A_k \) for all tip radii. Frequency at peak generally decreases with increasing \( A_k \). Comparing Fig.2 and Fig. 3 it is worth noting that the characteristic size of the experiment (tip radius) does have a considerable effect on the frequency at peak while it is irrelevant for the peak height.

**IV. CONCLUSION**

The results achieved in this work show the relevance of the frequency domain modeling allowing for anisotropic permeability. As the characteristic size of the experiment (tip radius) is showing a considerable effect on some of the experimental measures (frequency at peak of time shift) and a negligible effect on other measures (height of the time shift peak), an experimental procedure can be envisaged in which different tip radii are used so to identify both tissue elastic and permeability anisotropy of the superficial layers.

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**REFERENCES**


